MEASUREMENT OF HORIZONTAL BEAM SIZE USING SEXTUPOLE MAGNETS IN THE CESR STORAGE RING

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Understanding how changing the strength of sextupoles affects the beam of the Cornell Electron Storage Ring allows us to accurately measure the size of the positron beam bunch at the sextupole. There is no previous work of this kind, so it is integral in improving all particle accelerators. Data was collected from CESR by changing the strength, or k2 value, of a designated sextupole in intervals. The collected data was optimized using the CESRV program and then analyzed using two methods with the program PAW. Using these output graphs, the values were used to directly calculate the horizontal size of the beam at the designated sextupole. Using the single kick analysis seemed to be less accurate than the wave analysis, but both gave viable beam sizes. This method is proven to work, and it is accurate when compared to the current accepted beam size.

1 Introduction

The main goal of this project is to accurately derive the beam size using the sextupoles in the Cornell Electron Storage Ring, or CESR. We manually change the strength of designated sextupoles and run the data through a series of optimization and analysis programs. This outputs values that are used to directly calculate the beam size and its error at the designated sextupoles.
First introduced in 1967, Cornell University's particle accelerator has been integral in many major physics discoveries. Many changes have been made over time, which will be further discussed in a brief history of the Cornell Accelerator. The history is important in understanding the ideas behind the physics I will be discussing later.

1.1 Background Information

Cornell University's particle accelerator was first put to use in 1967. A variety of improvements, new detectors, and projects have been installed since it was first made. Some of these include the new CESR electron and positron storage ring, as well as its counterpart the CLEO detector. First proposed in 1975, CESR and CLEO had a slow start. It was designed to be a synchrotron light source and a positron electron collider. It wasn't until 1977 that the NSF approved the 20.6 million dollar project. Construction was finished only two years after, and the first electron beam was injected in the early morning of April 2nd, 1979. Shortly after, the CLEO detector was completed and research began. The CLEO detector was put out of use in 2008, and combined function multipoles were installed in its place in the south arc.
CESR has been in use since 1979, is 768 meters in circumference, and runs in the 3.5-12GeV energy range.

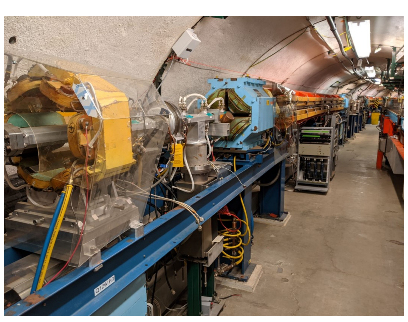


Figure 1: From left-to-right: Sextupole(Yellow) Quadrupole(Blue) Dipole(Red)

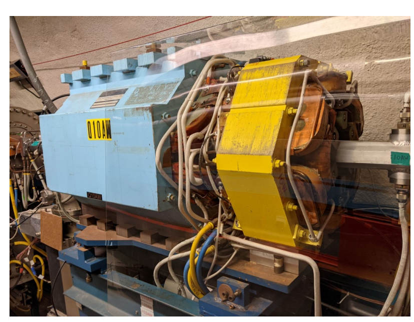


Figure 2: 10AW Sextupole Example

Particles are oscillated, revolved, and collided at relatavistic speeds to be researched. It utilizes three major

types of magnets, the dipole, quadrupole, and sextupole. There is a RF superconducting Cavity, and 100 Beam Position Monitors that we collect data from. The Superconducting RF cavities are used to accelerate particles back to relatavistic speeds, as they lose energy in the form of radiation as the beam gets bent around corners and re-focused. This project focuses on sextupole magnets specifically, but it is important to know how all three magnets interact to maintain a positron or electron bunch.
The storage ring is set up as a series of dipoles, then a quadrupole that is immediately followed by a sextupole. The dipole magnets serve to bend the beam bunch around turns so it can revolve around the storage ring. The quadrupole magnets serve to focus the beam, as it will naturally spread out over time. Sextupoles correct dispersion in the ring. As seen in Figure 1, sextupoles are found right after every quadrupole. This is because they are corrective magnets for the quadrupole error. Since particles come into the quadrupoles at different momentums, speeds, and positions, they change direction at different magnitudes. Sextupoles have specific magnetic fields that correct this and re-focus bunches, as seen in Fig. 3. More information on sextupole magnetic fields and calibration will be discussed in section 2.2.

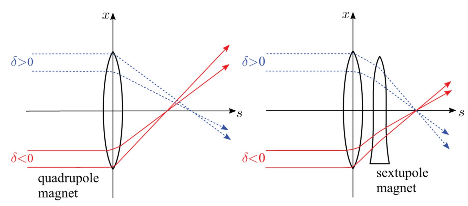


Figure 3: Sextupole Correction Example

1.2 Relevance

This project is the first of its kind, as there are no records of this being done in the past. This can be used to improve our understanding of the beam at the sextupoles. It not only improves the calibration of Cornell University's synchrotron, but it can be utilized to improve all other synchrotrons that use the same types of magnets. With future improvements to the project, the CESRV simulation model can be more accurate to how CESR functions, which will greatly improve the research of anyone using the online simulation.

2 Theory Background

2.1 Relevant Equations and Derivations

I will go over some of the relevant equations and their derivations in this section. The equations used to describe the effect of the magnetic field on the beam bunch are as follows.

The Lorentz Force equation describes how the magnetic field and velocity interact to create a magnetic force on a charge. There is no electric field in the storage ring, therefore that part of the equation is negligible and not included.

$$\vec{F} = q\,\vec{v} \times \vec{B} \tag{1}$$

The sextupole magnetic field can be determined by the magnetic force (using the Lorentz equation) and the offset of the beam from the magnets.

$$B_{\mathrm{y}}(\mathrm{x}) = B_{\mathrm{y}}^{''}\ \mathrm{x}^2 \tag{2}$$

The charge distribution is described by the integral shown. It's composed of the difference between the position of a charge and the center position, as well as the beam size.

$$q = q_0 \int_{-\infty}^{+\infty} \exp\left(\frac{(\mathrm{x}-\mathrm{x}_0)^2}{\sigma^2}\right) \mathrm{dx} \tag{3}$$

The combination of equations 2 and 3 gives us an expression for the average force of the sextupole magnetic field on the charge distribution. It is dependent on the overall beam size, also known as σ.

$$\langle F_{\mathrm{x}}\rangle_{\mathrm{x}_0,\sigma} \,\&q_0\ B_{\mathrm{y}}^{''} \int_{-\infty}^{+\infty} \mathrm{x}^2 \exp\left(\frac{(\mathrm{x}-\mathrm{x}_0)^2}{\sigma^2}\right) \mathrm{dx} \tag{4}$$

$$\&q_0\ B_{\mathrm{y}}^{''}\ (\mathrm{x}_0^2 + \sigma^2)$$

The derivation of the equation we used to calculate the beam size results is as follows:

$$\sigma^2 = \frac{4\tan(\pi Q)}{\beta}\frac{\mathrm{d}x}{\mathrm{d}k_2} - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2 + (k_2 l\frac{\mathrm{d}x}{\mathrm{d}k_2 l})^2\left(1 + \frac{\mathrm{d}k_2 l}{k_2 l}\right) \tag{5}$$

$$\sigma^2 = \frac{4\tan(\pi Q)}{\beta}\frac{\mathrm{d}x}{\mathrm{d}k_2 l} - (X_0)^2 = 2\frac{\mathrm{d}px}{\mathrm{d}k_2 l} - (X_0)^2 \tag{6}$$

$$\sigma^2 = 2\frac{\mathrm{d}px}{\mathrm{d}k_2 l} - (X_0)^2 \tag{7}$$

$$\sigma^2 = 2\frac{\mathrm{d}px}{\mathrm{d}k_2 l} - \left(\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}\right)^2 \tag{8}$$

The final equation in the derivation is used to measure the beam size in units of mm^2. The $\mathrm{d}k_1$l, $\mathrm{d}k_2$l, and dpx values are very important in understanding the calculations. The $\mathrm{d}k_1$l value, also known as the phase kick or quad kick, describes quantitatively how the sextupole corrects the transverse momentum change from the quadrupole multiplied by the sextupole length. Similarly, the dpx value is known as the dipole kick, or the orbit kick. The $\mathrm{d}k_2$l value describes the strength of the sextupole, k2, multiplied by the length of the sextupole, l. The k2 value is what we change to collect data that allows us to calculate the beam size.
The $\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l}$ value is most accurate from the tune analysis. It is derived from the following equation regarding the change in tune.

$$\frac{\mathrm{d}k_1 l}{\mathrm{d}k_2 l} = \frac{1}{\beta}\frac{\mathrm{d}\nu}{\mathrm{d}k_2 l} \tag{9}$$

The initial value for the beta function at the sextupole is β. We get this value from the tune analysis. The equation for $\mathrm{d}\nu$ is described below, where 390.1 is the revolution frequency in kHz, and $\mathrm{d}f$ is the change in tune in a revolution.

$$\mathrm{d}\nu = 2\,\frac{2\pi}{390.1}\,\mathrm{d}f, \tag{10}$$

The error calculations are also important in understanding the data and its accuracy. The following equation describes the relative error in the square beam size. An in-depth explanation of the error analysis can be found in section 4.5.

$$((\delta\sigma))^2 = \left(\delta\left(2\frac{dpx}{dk_1 l}\right)\right)^2 + \left(\delta\left(\frac{dk_1 l}{dk_2 l}\right)^2\right)^2 \tag{11}$$

To check the data and confirm its accuracy, the values for beta, eta, delta, and epsilon data are used. The design value for the beam size is compared to the calculated beam size from these values using the following equation. This has proven that the data we collected is fairly accurate to the design, so the data is usable.

$$(\sigma)^2 = \beta\epsilon + (\delta)^2(\eta)^2 \tag{12}$$

To check if the optimized and analyzed data is accurate, we use an emittance check. Re-arranging the above equation to solve for emittance allows us to use our calculated beam size to check the accuracy of the results. An in-depth explanation of the emittance checks can be found in section 4.5.

$$\epsilon = \frac{\left((\sigma)^2 - (\delta)^2(\eta)^2\right)}{\beta} \tag{13}$$

2.2 Sextupole Calibration

Inaccurate sextupole calibration has affected the results of this project significantly. According to a paper by Alexander Mikhailichenko, in 1998 7.36mm was cut off every pole of 74 of the 76 sextupoles. This has led to inaccuracies in the calibration since. There is currently around a 10% uncertainty in CESR due to inaccurate calibration. The source of this inaccuracy is calculated below.
CBN 98-2 provided a field integral estimate that gave the following results:

$$\int B_y dl = B_y l = 10.6553(T/m)$$

Where the length is 0.0100m for 10A of excitation current with an accuracy of 0.01%. This gives us a result of 10.6553±0.00106 T/m.

According to John Barley at Cornell Unviersity, CESR runs full scale in the control system at 16,000 CU at 12.5A. The sextupole length is 0.270m, so the k_2l value is 6.6549 m^{-2} for the CESR parameters. The k_2l per CU is 4.1593e^{-5} m^{-2}, and the k_2 per CU is 1.5292e^{-4} m^{-3}. This is the source of error, as the

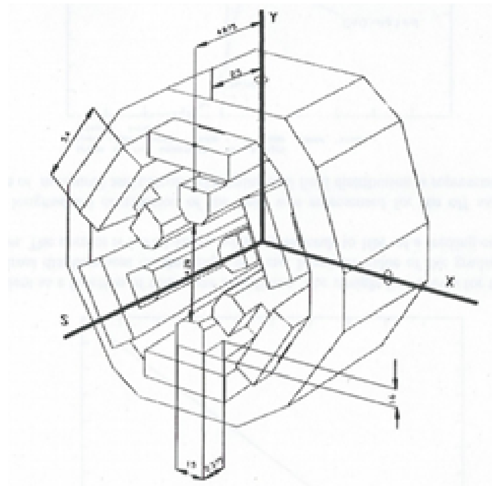


Figure 4: CESR Sextupole Dimensions in Inches

calculations we have done use $1.531e^{-4}$ m^{-3} instead. This was scaled in the database by 5.29GeV/6GeV and the original calculations were done using the speed of light as $3e^8$ m/s instead of $2.9979e^8$. All of these factors give an uncertainty of around 10%, which has skewed the results further. With better understanding of sextupole calibration in CESR and more accurate values, the beam size results can be improved as well as the online CESRV model.

3 Desctiption of Experiment

3.1 Data Collection

I was able to attend a data collection session on June 22nd. We were able to get measurements for 5 different sextupoles that night from the east and west arcs as seen in Figure 5. The data collection process is fairly simple. The CESR storage ring is run and data is collected at normal levels. The strength of the sextupole is the changing variable, so we first decrease the strength by -30,000 Computer Units. Three sets of data are taken at each level, and the strength is increased by +6,000 CU until +30,000 CU is reached. Then the next designated sextupole gets altered. Some of the information we get from the sextupoles include the orbit, phase, and tune measurements. The orbit is the position of the beam in the horizontal and vertical planes, measured in millimeters. This is what we are using to constrain the optimization process. This is further explained in section 4.2. The phase is the betatron oscillation function, which is another factor being used to constrain the optimization process. The betatron oscillation function is a steady oscillation of all particles in the beam around the equillibrium point in both the horizontal and vertical direction measured in degrees. The tune is the number of betatron oscillations in one revolution of the beam, multiplied by the revolution frequency in the horizontal and vertical planes respectively. This is measured in Hz. The kick is derived from this data, and it is the change in the transverse momentum of the beam. The two types of kick we find are the quad kick, or the phase kick, and the dipole kick, also known as the orbit kick.

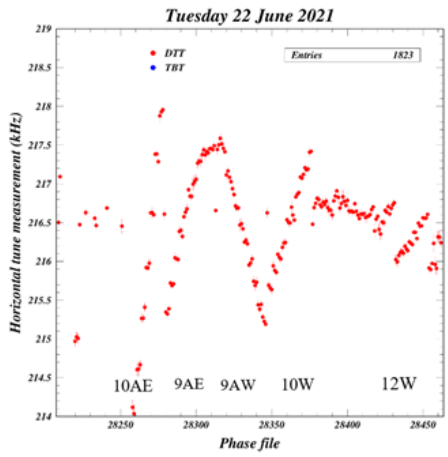


Figure 5: June 22nd Tune Data Collection

Figure 5 is a raw data sample from June 22nd. It shows the Horizontal Tune measurement for different sextupoles, which are labeled at the bottom. The phase file is the numbered file associated with each data point chronologically. As the strength increases, the tune, or number of horizontal betatron oscillations per second, changes linearly. Along the top of the 9AE/9AW lines, there is a slight parabolic shape that is due to some beam loss. However, most of the data behaves linearly, which is expected.

4 Presentation of Data

4.1 Primary Data

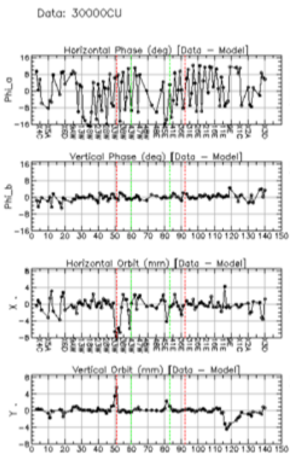
A in-depth explanation of how data is collected can be found in section 3.1, but for now I want to focus on interpreting the raw data and explaining how it is used.

Phase File No.	Sextupole No.	dk2(CU)	Beta	Phi	Eta
26550	10(10AW)	-30,000	33.80	16.912	-0.01
26551	10(10AW)	-30,000	34.03	16.913	0.01
26552	10(10AW)	-30,000	34.03	16.913	0.08
26553	10(10AW)	-1	40.26	16.736	-0.04
26554	10(10AW)	-1	40.22	16.742	-0.01
26555	10(10AW)	-1	40.18	16.714	-0.03
26556	10(10AW)	30,000	56.90	16.491	-0.10
26557	10(10AW)	30,000	58.68	16.464	-0.11
26558	10(10AW)	30,000	28.99	16.464	-0.10

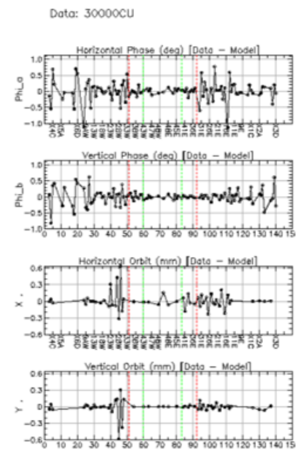
This is an example of the initial data from the output file rw_sxtwiss.dat, which is the organized version of the raw data. The first column, Phase File Number, refers to the numbered file associated with specific data. There are close to 4,000 of these files for all 74 sextupoles. There is more data in the file, such as the sextupole length, position, etc. All of these values are used in the optimization or analysis process. The dk2 value is the sextupole strength relative to 0. It is described in Computer Units, which CESR understands is the relative strength. The beta value describes the initial value for the beta function at the given sextupole. This value is important in multiple steps when calculating the horizontal beam size from sextupole strength. We find the average of all of the dk=0 beta values, which is a source of error later on. The phi and eta values are used during the emittance checks in section 5.1.

4.2 Optimization

The data is sent through a series of optimizations using two CESRV programs, one for each type of analysis. It is compared to the 'model' data, which is the online CESRV model. I manually added the June 22nd data and hand-tuned the fit ranges upstream and downstream of the sextupole to optimize the kick values and errors. The two different optimization programs are used the same way, but they use different data. One uses the wave data, and the other uses the single-kick data, which will be used for their respective analysis processes. We use two different data sets as some data has proven to be more accurate than others in each analysis process. More information can be found in the analysis sections.



(a) Pre-Optimization



(b) Post-Optimization

This outputs highly optimized data with very low errors that is used in the wave analysis. The optimization program functions by first clipping any extraneous data points, then running a series of mathematical optimizations that target certain data. The optimization compares the data to the online simulation model. It takes the online model and changes the orbit and phase values until it closely matches that of the collected data. This gives the program a better understanding of what happened during the physical data collection. It also gives us more in depth data than we were able to physically collect. It completes these optimizations until an error value called the Merit Function gets as low as possible. When the merit function value is to the magnitude of 10E-1, the optimization is complete. This is an example of sextupole 10AW. The scale for the post optimization, as seen in graph b, is much smaller than that of the pre-optimization. That means the online model has matched the data almost perfectly. The beta value, one of the major input values for optimization, has an uncertainty of around 10%. Future work can be done to minimize this error, which will get a more accurate beam size. Similarly, adding a beta constrained optimization to the process would greatly improve the accuracy of the data. This new data is used in the next step, analysis.

4.3 Wave Analysis

The data from the wave analysis process is giving promising beam size calculations. The wave analysis compares the different data files to the dk=0 file. The process follows three programs as follows. First, Wave_Analysis_and_Write_Output_Files serves as the optimization program as described in section 4.2. Waveloopall takes specific data from the output optimization files and writes it into a condensed pearl script. Finally using the PAW program, we run the anaw script which outputs graphs of the $\frac{dk_1 l}{dk_2 l}$ and $\frac{dpx}{dk_2 l}$ that we use directly to calculate beam size. The P2 value is very accurate for the $\frac{dpx}{dk_2 l}$, but the tune analysis gives a more accurate $\frac{dk_1 l}{dk_2 l}$ value as seen in the figures below. The more linear the $\frac{dk_1 l}{dk_2 l}$ graph is, the more accurate the P2 value is. We use the P2 value and error bar to calculate the beam size.

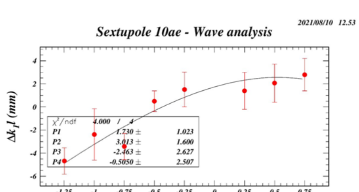


Figure 6: 10AE Wave Analysis $\frac{dk_1 l}{dk_2 l}$

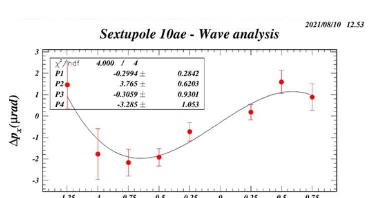


Figure 7: 10AE Wave Analysis $\frac{dpx}{dk_2 l}$

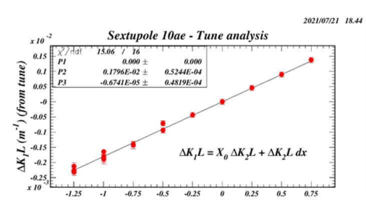


Figure 8: 10AE Tune Analysis $\frac{dk_1 l}{dk_2 l}$

An example of the output results we get from these P2 values is shown below. Using data from the wave analysis as well as the tune analysis, the beam size and its error can be calculated. σ^2 is the beam size

Sextupole	dk1l wave	error	dpx wave	Error	dk1l tune	error	sigma^2	error^2	Design Value^2
10AE	1.72	0.12	-3.82	0.12	1.826	0.019	4.31	0.90	1.32

squared, which is comparable to the design value squared. The design value is the expected beam size value from CESR as it was designed. The equations used to calculate these can be found in section 2.1. These calculated values are very close to the accepted design values, but there are still some errors in the wave analysis process that need to be worked on in the future. The data for all sextupoles using the wave analysis can be found in the table at the end of this paper.

4.4 Single Kick Analysis

There has been a lot of error in the single kick analysis so far. There are no current calculated points that are accurate to the accepted value for the beam size. Future work must be done on this to make it a feasible analysis process. This is an example of the single kick analysis from 10AW. Most other sextupole's data from this analysis process had negative σ^2 values that weren't able to be corrected by their error bars, so they were invalid.

Sextupole	**dk1l single**	**error**	**dpx single**	**Error**	**sigma^2**	**error^2**	**Design Value^2**
10AW	-5.013	0.062	12.82	0.20	0.51	.73	1.124

10AW is a good example for beam size error that can cause the beam size to be either positive or negative, as well as get the beam size closer to the target value. Most single kick results were negative and un-usable, so this process needs to be improved in the future to have useful data. The data for all sextupoles using the single kick analysis can be found in the table at the end of this paper.

4.5 Error Analysis

The error analysis is an important step in understanding the accuracy of the results. The error, uncertainty, and accuracy of the beam size are calculated simultaneously. As shown below, the error for σ^2, or beam size squared, is directly calculated.

$$((\delta sigma)^2)^2 = \left(\delta\left(2\frac{dpx}{dk_1 l}\right)^2\right)^2 + \left(\delta\left(\frac{dk_1 l}{dk_2 l}\right)^2\right)^2$$

The relative uncertainty is used to calculate the error in σ. As seen in the example from the wave analysis of sextupole 10AW, the relative uncertainty between 2.21mm^2 and 0.92mm^2 in the error is about 40% in sigma^2. Therefore there must be a 20% relative uncertainty in σ. Since the square root of 2.21 is 1.49 mm, the 20% relative error to that is 0.30mm.

5 Results

5.1 Resultant Data, Error Analysis, and Checks

The overall results for this project are fairly accurate given the amount of time it was worked on. The table below includes beam size squared values and errors from the wave analysis for some of the sextupoles. It includes a column for the accepted beam size squared, which is what the beam size is designed to be. It also includes the beam size calculated from the emittance, beta, eta, and delta values squared. These are derived from the phase measurements that are made during the data collection. It is the beam size from CESR currently, not the design value. They are similar though. The phase beam size is calculated using equation 12 in section 2.1. The following table includes the emittance checks. This is another way

Sextupole No.	**Beam Size^2**	**Error**	**Beta**	**Emittance**	**Eta**	**Delta**	**Phase Beam Size^2**	**Accepted Beam Size^2**
10AW	-1.72	0.25	40.27	2.66e-8	-0.015	8.31e-4	1.07	1.12
12W	3.38	0.19	20.47	2.66e-8	1.71	8.32e-4	2.56	2.47
34W	40.19	3.32	47.23	2.95e-8	1.77	8.21e-4	3.50	3.13
42E	3.11	0.18	24.08	2.84e-8	1	8.32e-4	1.38	1.23
12E	11.86	0.59	24.6	3.27e-8	2.01	8.25e-4	3.55	2.47
10AE	4.31	0.91	46.15	2.81e-8	-0.22	8.25e-4	1.33	1.32
9AE	-9.68	0.21	20.98	2.83e-8	-0.18	8.25e-4	0.62	0.61
9AW	1.01	0.59	17.96	2.84e-8	-0.02	8.24e-4	0.51	0.49
10W	-17.80	2.83	15.82	2.83e-8	-0.01	8.24e-4	0.45	0.42
12W	2.31	0.39	22.53	2.82e-8	1.71	8.24e-4	2.62	2.47

we check the accuracy of the calculated beam size. This method removes excess variables and allows us to confirm weather or not the calculated beam size was accurate by checking it against the emittance found using the accepted beam size. This has confirmed that the results are still somewhat inaccurate. The given emittance uses the same beta, eta, and delta values as the table above. The N/A values are due to the negative σ^2 value. The accepted emittances should be roughly the same according to the design. The calculated emittance values are mostly off by a factor of 10, which means the calculated σ^2 is incorrect for the values that are inaccurate to that magnitude.

Sextupole No.	Calculated Beam squared	Given Beam	Calculated Emittance	Given Emittance
10AW	-1.723	1.124	N/A	2.66E-8
12W	3.3819	2.465	6.64E-8	2.66E-8
34W	40.187	3.133	8.06E-7	2.95E-8
42E	3.1051	1.232	1.00E-7	2.84E-8
12E	11.8599	2.465	3.70E-7	3.27E-8
10AE	2.2144	1.323	N/A	2.81E-8
9AE	-9.682	0.608	5.63E-8	2.84E-8
9AW	1.012	0.49	N/A	2.83E-8
10W	-17.799	0.423	1.42E-8	2.82E-8
12W	2.305	2.465	1.42E-8	2.82E-8

5.2 Errors and Future Work

Using more accurate beta values, sextupole calibrations, and offsets can allow us to compare more data in the future. Currently there are only 10 measurements being compared to the k2=0 values, but we can compare every measurement to one another with more accurate values. The beta values need to be improved as well as the dk2/dcu values, is the sextupole calibration. We find the average of all of the dk=0 beta values, which is one source of error for the beta value. In a trial calculation, changing the dk2/dcu by around 10% and the beta value slightly, we are able to get much more accurate beam size values. This will be the next step in the project. When this process becomes more automized and accurate, we can greatly improve the online cesrv model which will improve the research of everyone using the online simulation.

6 Conclusion

While there are still many improvements to be made, the project was an overall success. This work produced some of the first accurate beam sizes with reasonable errors using this method. Far more accurate beam sizes can result from more accurate data and improved optimization methods in the future. This project will be the base for much more detailed work in the future.

7 Acknowledgements

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Table 1: **August 13, 2021 - Full data table for both Wave Analysis and Single Kick Analysis**

Sextupole No.	**Wave** $\frac{dk_1l}{dk_2l}$	**Error**	**Wave** $\frac{dpx}{dk_2l}$	**Error**	**Tune** $\frac{dk_1l}{dk_2l}$	**Error**	σ^2	σ^2 **Error**	**Single Kick** $\frac{dk_1l}{dk_2l}$	**Error**	**Single Kick** $\frac{dpx}{dk_2l}$	**Error**	σ^2	σ^2 **Error**
10AW	5.70	0.29	13.45	0.11	-5.35	0.011	1.723	0.247	-5.013	0.062	12.82	0.20	0.51	0.73
12W	0.346	0.084	1.767	0.093	0.39	0.015	3.38	0.19	0.302	0.024	-0.140	0.046	-0.37	0.09
34W	17.28	1.41	56.79	1.41	-8.57	0.10	40.187	3.32	-8.59	0.12	-20.36	1.83	-114.51	4.20
42E	-0.99	3.42	3.493	0.086	-1.97	0.017	3.11	0.18	-1.240	0.054	0.47	0.078	-0.591	0.21
12E	8.02	6.15	6.51	0.29	1.078	0.052	11.86	0.59	0.82	0.34	0.11	0.068	-0.45	0.57
10AE	1.57	0.83	2.72	0.45	1.80	0.052	2.21	0.92	1.74	0.12	-1.10	0.28	-5.23	0.69
9AE	1.80	0.16	-3.95	0.10	1.337	0.019	-9.68	0.21	0.782	0.028	-0.923	0.036	-2.46	0.08
9AW	-1.95	0.05	2.79	0.29	-1.724	0.027	1.01	0.59	-0.998	0.035	1.99	0.11	2.99	0.22
10W	3.15	1.41	-7.60	1.41	1.614	0.025	-17.80	2.83	1.10	0.024	-2.42	0.15	-6.05	0.30
12W	2.51	0.61	1.61	0.19	-0.15	0.014	2.31	0.39	-0.127	0.012	-0.563	0.039	-1.14	0.08